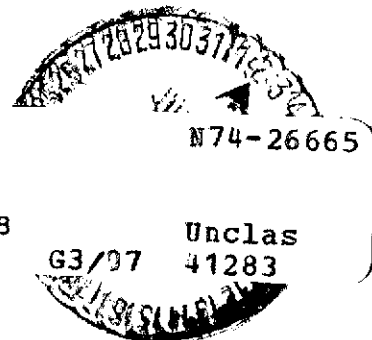|up

# TECHNICAL

# RESEARCH

# REPORT

FINAL TECHNICAL REPORT

STUDY OF OPTIMUM METHODS OF
OPTICAL COMMUNICATION

National Aeronautics and Space Administration

Grant No. NGR 21-002-237
Suppl. 1

# DEPARTMENT OF
# ELECTRICAL ENGINEERING

## UNIVERSITY OF MARYLAND

**COLLEGE PARK, MARYLAND 20742**

PRICES SUBJECT TO CHANGE

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FINAL TECHNICAL REPORT

STUDY OF OPTIMUM METHODS
OF OPTICAL COMMUNICATION

National Aeronautics and Space Administration

Grant No. NGR 21-002-237,
Suppl. 1

Principal Investigator
Dr. Robert O. Harger

Department of Electrical Engineering
University of Maryland
College Park, Maryland 20742

# 1. INTRODUCTION

The subject of the proposed research was the study of optimum methods of optical communication accounting for the effects of the turbulent atmosphere and quantum mechanics, both by the semi-classical method ("structured approach") and the full-fledged quantum theoretical model ("unstructured approach"). The proposed research was a continuation and extension of research accomplished and in progress with the support of National Aeronautics and Space Administration Grant NGR-21-002-0237.

The proposed work continued a concerted effort to apply the techniques of communication theory to the novel problems of optical communication by a careful study of realistic models and their statistical descriptions, the finding of appropriate optimum structures and the calculation of their performance and, insofar as possible, comparing them to conventional and other suboptimal systems. In this unified way the bounds on performance and the structure of optimum communication systems for transmission of information, imaging, tracking, and estimation can be determined for optical channels.

The three principal areas of proposed investigation were (1) a continuation of the earlier work on communication through the turbulent atmosphere (processing of images sequences-obtained in nonturbulent channels initially; structure and error performance of optimum demodulators of analog modulated waveforms transmitted through the lognormal turbulent channel), (2) performance with more complicated detector models using the semiclassical method of quantum mechanics (inclusion of effects of thermal noise, random channel and filtering), and (3) quantum communication

theory quantum estimation theory: maximum likelihood estimation, sufficient estimators; quantum detection theory: quantum S-games; perturbation solution of optimum detection operator equation).

## II WORK ACCOMPLISHED AND IN PROGRESS UNDER GRANT

The Grant has centrally contributed to establishing a large and active research group that comprises a major effort of the Department in research at the doctoral level. A seminar in optical communications has been held several semesters, as many as a dozen enrolled. A special topics course in optical communication is to be offered next year.

Work accomplished (i) The problem of selecting optimum binary signals for communication over a Poisson channel was studied under the minimum probability of error criterion. Such results have direct application to optical communication systems. In order to study this problem for relatively general signal sets, a Gaussian approximation for the distribution of a certain test statistic was made and its range of validity checked. The problem was then solved for symmetric intensity sets with direct detection and the resulting performance calculated and compared with that for other signal sets that would be reasonable choices.

The technique was applied to coherent heterodyne detection: the resulting signal design problem was shown to be the same as that for the "classical white, Gaussian noise channel."

Direct detection and coherent heterodyne detection were compared with rather interesting results that determined when one was preferable to the other.

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All these results are of direct significance to optical communication practice. The results were presented at the annual (1971) Princeton Conference on Information Sciences and Systems.

(ii) The optimum processing (likelihood functional) was found for a set of M images $\{Z_m = S_m + Y + N_m\}$, each the sum of a member $S_m$ of a signal sequence $\{S_m\}$, due to an object to be detected and its parameters estimated, a sample function $N_m$ of a noise field $\{N_m\}$, and a sample function Y of a common background field $\{Y\}$. The noise fields $\{\{N_m\}\}$ were independent, zero mean, white Gaussian fields and all independent of the background field $\{Y\}$; the latter was assumed to be either (i) completely unknown or of known mean and covariance functions with (ii) a certain fluctuation property or (iii) Gaussian. Three equivalent forms of the optimum processing were found: (i) a summation of generalized matched filterings of the images; (ii) a summation of matched filtering of certain generalized differences of the images; (iii) a summation of "estimator-correlator" type filterings. The detection performance and optimum signal/image selection under the Neyman-Pearson criterion was given and the singularity of the $(\{\{N_m = 0\}\}$ and $M > 1)$ case noted. It was shown that optimum processor and signal design can completely eliminate any effect of the background on detectability $(M > 1)$. The Cramer-Rao lower bound for the signal parameter estimate's mean-square error was given along with an example; optimum signal/image selection in the single parameter case was discussed. These results were published.

(iii) Some work on the complex representation of functions and

processes, whose interest is not confined to optical communication, was completed and submitted for publication.

(iv) The initial results on quantum S-games was presented in a lecture at the Center for Naval Analysis [42].

(v) Three doctoral theses have been completed with or (in one case) in conjunction with Grant supported work:

Michael Tacey, "Optimum Photon Counting Receivers using Quantum

Mechanical Models ."

Shiraz Bhanji, "Binary Communication with a Filtered Poisson Process

in White Gaussian Noise."

Koji Kurimoto, "Optimum Filtering and Detection for a Doubly Stochastic

Shot Noise process in White Gaussion Noise."

(1) The study of optimum phase demodulation and its peformance for a field received through a turbulent channel, began under the initial Grant has been continued.

(ii) The solution of the M-ary quantum detection problem in tractable special cases and the study of quantum estimation is the subject of a doctoral thesis (M. Kim).

Related work - (i) The results on the optimum reception of random fields has suggested a new way to study the problem of phase errors in received signals. This problem arises fundamentally, of course: the model we are studying applies at microwave frequencies but may be useful at optical frequencies. This study is the subject of a doctoral thesis (R. Glock).

(ii)  The Principal Investigator organized a Workshop in Optical

Communication Systems jointly sponsored by the University of Maryland

and the National Science Foundation held in January 1972 which brought

togehter about forty experts in the field to assess the current state in

certain areas, ascertain the outstanding problems and their priorities,

attempt to determine how universities might best contribute to this new

area, and assist the National Science Foundation in initiating a program

in the optical communication systems area.  The Workshop report, edited

by the Principal Investigator, was distributed to more than 175 universities.

# III ABSTRACTS OF WORK ACCOMPLISHED

## UNDER GRANT

A.  Papers published

1.  "On optimum binary signals for Poisson channels," 1971 Princeton
    Conference on Information Sciences and Systems (abstract in
    Proceedings).

2.  "Optimum Processing of a Signal Sequence in an Image Set with
    Additive Noise and Common Background," IEEE Transactions
    on Aerospace and Electronic Systems, March, 1972.

3.  "The formulation and examples of quantum S-games," presented
    at the Center for Naval Analysis in lecture series on detection
    theory and its applications, October, 1971.

B.  Paper submitted for publication

4.  "On the mapping associated with the complex representation of
    signals and processes," submitted to IEEE Trans. AES,
    October, 1971.

C.  Doctoral theses completed

5.  "Optimum photon counting receivers using quantum-mechanical
    models," M. P. Tracey, May, 1971.

6.  "Optimum filtering and detection of a doubly stochastic shot noise
    process in white Gaussian noise," K. Kurimoto, May, 1972.

7.  "Binary communication with a filtered Poisson process in white
    Gaussian noise," S. A. Bhanji, May, 1972.

D.  Work in progress

8.  "Optimum demodulation of phase modulated wave-forms transmitted
    over a turbulent channel," being prepared for publication in 1972.

9.  "On solutions of the optimum detection operator equation in certain
    cases and approximations," doctoral thesis topic, M. Kim, 1972.

ON OPTIMUM BINARY SIGNALS
FOR POISSON CHANNELS*

R. O. Harger and K. Kurimoto
Department of Electrical Engineering
University of Maryland
College Park, Maryland 20742

## ABSTRACT

We consider the problem of optimum binary signalling under the minimum probability of error (Pe) criterion for an optical communication channel described by time-varying Poisson processes. Previous work established the optimum receiver and determined the optimum signals under several criteria and generally in special cases. In order to discuss signal optimization over a larger class of signals, the relevant test statistics are assumed Gaussian: the range of validity of the approximation is established.

For direct detection, symmetric intensity sets are defined: a member of such a pair is a reflection about an arbitrary point of a certain periodic extension of the other. We show that for symmetric intensity sets the minimum Pe is achieved in the limit by a sequence of disjoint, constant signals of support tending to zero. In the high background noise situation we show that the signal design problem under the minimum Pe criterion is the same as that under the maximum SNR criterion of Reiffen and Sherman.

For coherent heterodyne detection, we show that for equally-likely, equal energy signals, the signal design problem is the same as that for known signals in the white Gaussian channel.

Assuming equal energy (E) signals and an average signal count much greater than unity, we show that direct detection using the optimum symmetric intensity set is inferior to coherent heterodyne detection using antipodal signals. However, direct detection with "ON-OFF" signal set (the "ON" signal of energy 2E) can be superior to coherent heterodyne detection using equal energy (E) antipodal signals, provided the ratio of average signal count to noise count is sufficiently large.

# OPTIMUM PROCESSING OF A SIGNAL SEQUENCE IN AN IMAGE

# SET WITH ADDITIVE NOISE AND COMMON BACKGROUND

## Abstract

The optimum processing (likelihood functional) is found for a set of $M$ images $\{Z_m = S_m + Y + N_m\}$, each the sum of a member $S_m$ of a signal sequence $\{S_m\}$, due to an object to be detected and its parameters estimated, a sample function $N_m$ of a noise field $\{N_m\}$, and a sample function $Y$ of a common background field $\{Y\}$. The noise fields $\{\{N_m\}\}$ are independent, zero mean, white Gaussian fields, and all independent of the background field $\{Y\}$; the latter is assumed to be either 1) completely unknown or of known mean and covariance functions with 2) a certain fluctuation property or 3) Gaussian. Three equivalent forms of the optimum processing are found: 1) a summation of generalized matched filterings of the images, 2) a summation of matched filtering of certain generalized differences of the images, 3) a summation of "estimator-correlator" type filterings. The detection performance and optimum signal/image selection under the Neyman-Pearson criterion is given and the singularity of the $(\{\{N_m = 0\}\}$ and $M > 1)$ case noted. It is shown that optimum processor and signal design can completely eliminate any effect of the background on detectability $(M > 1)$. The Cramer-Rao lower bound for the signal parameter estimates mean-squared error is given along with an example; optimum signal/image selection in the single parameter case is discussed.

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# The Formulation and Examples of
# Quantum S-games

## Abstract

The extension of S-games to include quantum mechanical measurements is formulated and some illustrative examples given. The insight obtainable by the S-game geometry is used to find minimax and Bayes solutions to quantum S-games. The optimum decision rule for minimum probability of error quantum detection is given for simple examples of commutative and unitarily related density operators in the binary case.

# ON THE MAPPING ASSOCIATED WITH THE
# COMPLEX REPRESENTATION OF FUNCTIONS
# AND PROCESSES

## ABSTRACT

The mapping between function spaces that is implied by the representation of a real "bandpass" function by a complex "lowpass" function is explicitly emphasized. The discussion is extended to the representation of random processes where the mapping is between spaces of random processes.

<div align="center">ABSTRACT</div>

Title of Thesis: Optimum Photon Counting Receivers
Using Quantum-Mechanical Models

Michael Paul Tacey, Doctor of Philosophy, 1971

Thesis directed by: Dr. Robert O. Harger, Associate Professor
of Electrical Engineering

For M-ary signal detection at optical frequencies, the structure,
implementation and performance in terms of average probability of
error of the Optimum Photon Counting Receiver (OPCR) using quantum-
mechanical field models are investigated. Attention is restricted to
four modulation schemes that give rise to received signal fields which
can be represented in an orthonormal cavity representation having dis-
joint mode index sets for different signals and, therefore, give rise
to commuting density operators. Three channel models are used: the clear
space channel and the atmospheric turbulence channel with either Gaussian
or log-normal turbulence; additive background noise is assumed present
with each.

Previous work in this area has been directed towards the investi-
gation of Optimum Quantum-Mechanical Receivers (OQMR) and optimum receivers
obtained using classical models for the received fields. These receivers
are compared where applicable with the OPCR described here. When the den-
sity operators are diagonal in the number representation, the OQMR is the
same as the OPCR. An easily implemented Suboptimum Photon Counting Receiver
(SPCR) that compares sums of photon counts is investigated in the clear
space channel and for incoherent light with equal photon counts per mode
is found to be the same as the OPCR.

It is shown that in the clear space channel with no background noise, the performance of the OPCR for incoherent light asymptotically approaches the performance of the OPCR for coherent light as more cavity modes are excited. The form of the density operator for coherent signals in the Gaussian turbulence channel is shown to have a form similar to that of a coherent field plus an incoherent field in the clear space channel. In order to obtain performance results for both coherent and incoherent signals in the log-normal turbulence channel, statistically independent mode coefficients are assumed. It is shown that communication with coherent and incoherent light in the atmospheric turbulence channels can approach that of coherent light in the clear space channel depending on the background noise, signal energy and the availability of statistically independent cavity modes.

# ABSTRACT

Title of Thesis:    Optimum Filtering and Detection for a Doubly Stochastic

shot noise process in white Gaussian noise.


For the problem of optimum filtering and binary detection of signals

at optical frequencies, the structure, implementation and performance in

terms of average probability of error of the optimum and/or suboptimum

receiver in which the observed data is the superposition of shot noise process

and white Gaussian thermal noise are investigated. Attention is restricted

to the direct detection of a single path optical communication channel where

the received field is assumed to be the signal field degraded by atmospheric

turbulence with additive background noise.

Assuming the semiclassical methods of quantum mechanics and using

the theory of point process as it is asserted that, with small background

radiation, the shot noise process can be considered to be generated by an

inhomogeneous conditional Poisson inpulse process, with a rate parameter

of which is a function of a set of parameters describing the state of turbulence.

Using the Ito representation for observed data, shot noise process and

for the state of turbulence which is assumed to be a Gauss Markov process, the

basic equation which described the time evolution of the posteriori statistics

of shot noise process as well as those of the state of turbulence are derived

in terms of a stochastic differential form of the conditional joint characteristic

function. With this equation, the quasi optimum receive structure is specified

in its most general form.

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In order to evaluate the quality of a specified receiver a suboptimum receiver for a frozen atmosphere, log normal fading channel is obtained with justification. Under the condition of small correlation time roughly the inverse bandwidth of photodetector of the receiver and large sampling interval of the observed process and large detection time, the approximate probability of error for this receiver is obtained in terms of the solution of a simple Fokher Plank equation and also plotted, which shows that the effect of thermal noise is significant.

Previous work in this area has been mostly directed toward the investigation of optimum photoelectron counting receiver, the probability of which is also plotted assuming the same signals. Though it expected that the probability of error performance of the suboptimum receiver could approach to that of the photon counting receiver, this cannot be observed because of an assumed approximation.

# ABSTRACT

Title of Thesis:   Binary Communication with a Filtered Poisson
Process in white Gaussian Noise.

Shiraz G. Bhanji, Doctor of Philosophy, 1972.

Thesis directed by: Dr. Robert O. Harger, Associate Professor
of Electrical Engineering.

For detection of a filtered Poisson process in zero-
mean white Gaussian noise, optimum receiver structures,
optimum signal design, and receiver performance in terms of
the probability of error are investigated. The problem has a
direct application to signal detection at optical frequencies.
The output of a direct detection receiver can be represented
as a sum of a conditional filtered Poisson process, whose
rate parameter is proportional to the intensity of the signal,
and zero-mean white Gaussian thermal noise. Considerable
work has been done in this area but most of it has been
directed towards the two limiting cases of the above i.e.
representing the output of the direct detection receiver
either as a Poisson process or as a Gaussian process.

The likelihood functional for the general optimum
receiver is obtained by various methods and found to consist
of an infinite sum. From the general likelihood ratio the
closed form expressions are obtained in the two special
cases of i) a threshold receiver and ii) a low intensity
receiver.

The optimum threshold receiver derived by the state-variable approach for a single-state equation is compared with that derived by a straight-forward Taylor series expansion of the conditional likelihood functional and found to be identical. The optimum threshold receiver is also compared with other known receiver structures and to a first approximation the log-likelihood functional of the threshold receiver is found to be identical to that for a Gaussian signal in zero-mean white Gaussian noise. Assuming the test statistic to be Gaussian the performance in terms of the probability of error is evaluated for the threshold case.

For an infinite and a specific finite bandwidth detector and a given signal energy, minimum probability of error in the threshold case is achieved by a sequence of constant intensity pulses whose support tends to zero.

Assuming the test statistic for the low intensity receiver to be Gaussian, the performance of the receiver is evaluated in terms of the probability of error. For the low intensity case, with the assumption of "smooth" intensity modulation (smooth with respect to the impulse response of the detector) and symmetric signals of constant energy, maximum divergence is achieved by a sequence of disjoint pulsed intensities of support tending to zero. Under similar assumptions as above minimum probability of error is reached by disjoint pulsed signals of intensity proportional to the noise and detector parameters.

Except in the threshold and low intensity cases, closed form expression for the optimum receiver is difficult to obtain as shown by three different approaches (i.e. Taylor series expansion, state-variable approach, and classical method). Also the more promising state-variable approach is shown to reduce to the straight-forward Taylor series expansion of the conditional likelihood ratio when the conditional estimates are actually solved for.

Since this whole effort deals only with the problem of signal detection, a natural extension of the work is suggested in signal estimation including parameter estimation and maximum likelihood estimation of analog modulated signal intensity.

## Optimum Demodulation of Phase-modulated Waveforms
## Transmitted over a Turbulent Channel

### Abstract

The likelihood functional and likelihood equation necessary for the

maximum likelihood estimate of a phase modulation received over a noisy,

spatially turbulent channel is derived and shown to be closely related for

unknown, Gaussian, and limiting cases of the lognormal channels. The

likelihood equation is the same as that for known turbulence except that

the maximum likelihood estimate of the turbulence-perturbed received

signal field is used. The linearized likelihood equation is studied: the

conditions for linearization noted along with "diversity" improvement of

the optimum receiver, the natural relation to certain linear estimation

problems noted, and the resulting mean-square error of estimation.

# On Solutions for the Optimum Quantum Detection Operator In Certain Cases and Approximations

## Abstract (Partial)

A perturbation solution for the optimum (minimum probability of error) quantum detection operator in the binary case is calculated. The results are applied to the detection of a weak, coherent signal in thermal noise and the probability of error and optimum observable found. The realization of the optimum observable is studied: it appears to be only approximately realizable by a combination of photon counting and heterodyning.